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JHEP06(2009)053

(http://iopscience.iop.org/1126-6708/2009/06/053)

Published by IOP Publishing for SISSA
Received: *May 14, 2009*
Accepted: *May 31, 2009*
Published: *June 16, 2009*

Aspects of U-duality in BLG models with Lorentzian metric 3-algebras

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Abstract: In [1], it was shown that BLG model based on a Lorentzian metric 3-algebra gives Dp-brane action whose worldvolume is compactified on torus T^d ($d = p - 2$). Here the 3-algebra was a generalized one with $d + 1$ pairs of Lorentzian metric generators and expressed in terms of a loop algebra with central extensions. In this paper, we derive the precise relation of the coupling constant of the super Yang-Mills, the moduli of T^d and some R-R flux to VEV's of ghost fields associated with Lorentzian metric generators. In particular, for $d = 1$, we derive the Yang-Mills action with θ term and show that $\mathrm{SL}(2, \mathbf{Z})$ Montonen-Olive duality is realized as the rotation of two VEV's. Furthermore, some moduli parameters such as NS-NS 2-form flux are identified as the deformation parameters of the 3-algebras. By combining them, we recover most of the moduli parameters which are required by U-duality symmetry.

Keywords: D-branes, M-Theory, String Duality

ArXiv ePrint: 0905.1445

doi:10.1088/1126-6708/2009/06/053

Contents

1 Introduction and summary

Recently, Bagger, Lambert [2] and Gustavsson [3] found that a certain class of Chern-Simons matter system can have maximal supersymmetry in $2+1$ dimensions and that it may describe the multiple M2-branes. Their action is distinctive in that the gauge symmetry is based on a new mathematical framework, Lie 3-algebra. However, it was soon realized that their constraints on the algebra are too restrictive that the only allowed 3-algebra is so-called $\mathcal{A}_4$ algebra which describes the two M2-branes [4].

For the description of larger number of M2-branes, many studies have been made to generalize the BLG framework.[1] The first interesting example was found by three groups [6–8] which is based on the 3-algebra with a pair of Lorentzian metric generators u, v and arbitrary Lie algebra generators T^i, such that

$$\begin{aligned} &[u, T^i, T^j] = i f^{ij}{}_k T^k\,, \qquad && [T^i, T^j, T^k] = -i f^{ijk} v\,, \\ &\langle u, v\rangle = 1\,, && \langle T^i, T^j\rangle = \delta^{ij}\,, \end{aligned} \tag{1.1}$$

where we keep only the nonvanishing 3-commutators and metric components. While the components associated with the generators u, v become ghosts, they can be removed by a

[1] Apart from the examples mentioned below, there is also an example based on the 3-algebra with Nambu-Poisson bracket [5]. This algebra describes the infinite number of M2-branes and realizes the worldvolume theory of a single M5-brane in the C-field background on a 3-manifold where Nambu-Poisson bracket is equipped.

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new kind of Higgs mechanism proposed by [9]. After the ghosts are removed, the Chern-Simons matter system is reduced to the ordinary super Yang-Mills system which describes multiple D2-branes. Then many studies are undertaken on this Lorentzian BLG model [1, 10, 11]. However, since the correspondence is too exact, the model was realized to be too simple to describe the full M2-brane dynamics.

Soon after, another $2+1$ dimensional Chern-Simons matter system with $\mathrm{U}(N)\times\mathrm{U}(N)$ gauge symmetry was proposed [12]. While it lacks the manifest $\mathcal{N}=8$ supersymmetry, it has many attractive features such as the brane construction, AdS/CFT correspondence, and an intimate relation with the integrable spin chain. In particular, it gives a description of M2-branes when the coupling constant N/k (k is the level of Chern-Simons term) is large, namely in the nonperturbative region. In the perturbative range $N/k \ll 1$, it describes a system where the transverse space of M2-branes becomes $\mathbf{C}^4/Z_k$ with $k \gg 1$, which is getting closer to D2-branes in type IIA string theory.

The models based on the Lorentzian metric 3-algebras [6–8] which was later generalized in [1, 11] by including more Lorentzian metric generators (in the following, we call it 'L-BLG model' in short). Nevertheless, they still enjoy unique advantages that they keep $\mathcal{N}=8$ supersymmetry as well as SO(8) R-symmetry. Of course, M-theory requires such symmetry explicitly, so we believe that L-BLG models will be able to provide some nontrivial information on M-theory.

In this paper, as one of such examples, we examine how U-duality [13] is realized in L-BLG models.[2] It is based on a work [1] where a description of M-theory on higher dimensional torus T^{d+1} was given by generalization of 3-algebra with more Lorentzian metric pairs, say (u_A, v^A) $(A=0,1,\ldots,d)$.[3] As a generalization of the original model, we have $d+1$ pairs of the ghost fields associated with each (u_A, v^A). By choosing the structure of 3-algebra carefully, it has been shown that such ghost modes can be removed and the system becomes unitary as in the original model. Here, for the simplicity of the arguments, we will work with a *gauged* version of L-BLG model [15, 16] where the removal of the ghosts is exact. In this Higgs mechanism, one has to assign VEV's to these ghost fields as

$$X^I_{u_A} = \lambda^{IA}\,, \qquad \vec{\lambda}^A \in \mathbf{R}^{d+1} \subset \mathbf{R}^8\,. \tag{1.2}$$

These VEV's $\vec{\lambda}^A$, in turn, describe how the transverse directions $\mathbf{R}^8$ are compactified on T^{d+1}. In other words, the Higgs mechanism of L-BLG model produces the Kaluza-Klein mass associated such compactification. In [1], it was shown that L-BLG model gives a super Yang-Mills system whose worldvolume is a flat T^d bundle on $\mathcal{M}$, where $\mathcal{M}$ is the worldvolume of BLG model. In the section 2 of this paper, we perform a more detailed analysis with general $\vec{\lambda}^A$ and determine the precise relation of the coupling constant, moduli

[2]The Montonen-Olive duality in ABJM context was discussed in [14]. In their study, the coupling constants of the super Yang-Mills are restricted to depend only one real variable. In our case, there is no such limitation.

[3]Somewhat similar analysis was made on the generalization of the Lorentzian metric [11]. Their analysis was limited to the finite dimensional cases and does not include the 3-algebra which is the main focus of this paper.

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of the torus T^d, and some R-R flux on Dp-brane worldvolume theory to VEV's $\vec{\lambda}^A$ of L-BLG model.

These parameters are sufficient to fix all the moduli of D3-branes theory that corresponds to $d = 1$ case. Indeed, in the section 3, we argue that the action thus derived reproduces the complete 4-dimensional super Yang-Mills action with θ term. In particular, Montonen-Olive SL$(2, \mathbf{Z})$ duality [17] is realized by the rotation of the VEV's,

$$\vec{\lambda}'^A = \Lambda^A{}_B \vec{\lambda}^B , \qquad \Lambda^A{}_B \in \mathrm{SL}(2, \mathbf{Z}) . \tag{1.3}$$

While we do not claim that we prove the duality symmetry, the simplicity of the realization is nevertheless remarkable. For $d > 1$, it is natural to guess that the SL$(d + 1, \mathbf{Z})$ part of the U-duality transformation is described by the change of the basis as (1.3) where $\Lambda \in \mathrm{SL}(d + 1, \mathbf{Z})$. We note that U-duality group is give by a product SL$(d + 1; \mathbf{Z}) \bowtie O(d, d; \mathbf{Z}) =: E_{d+1(d+1)}(\mathbf{Z})$, where the symbol $\bowtie$ denotes the group generated by the two non-commuting subgroups (see, for example, a review article [18]). The $O(d, d; \mathbf{Z})$ part represents the T-duality symmetry. In our formulation, it is realized by the T-duality relation by Taylor [19].

Actually, for $d > 1$, the moduli parameters obtained from Higgs VEV's $\vec{\lambda}^A$ are not enough to realize full U-duality group. The description of U-duality covariant parameters for super Yang-Mills system is given in the context of BFSS matrix theory [20, 21]. One of such missing parameters is the NS-NS 2-form flux. We know already that this parameter can be included in the theory by the redefinition of the 3-algebra [1]. As d getting larger, we need more kinds of R-R flux also. We give some argument that these extra parameters will be obtained by changing 3-algebra further, possibly by including contributions of Nambu-Poisson algebra as [5].

2 Dp-brane action from BLG model with moduli parameters

In this section, we perform more detailed analysis of L-BLG model which is described in section 5 of [1]. The novelty of the following analysis is to introduce general VEV's for the ghost fields which give rise to the nontrivial metric for the torus T^d and an extra coupling constants which are related with some R-R flux on Dp-brane. The action after Higgs mechanism is summarized in section 2.5. We give also more careful explanation of the compactification mechanism and the geometry of the Dp-brane worldvolume.

2.1 BLG Lagrangian and 3-algebra for Dp-brane

The original BLG action is written as [2]

$$\begin{aligned}
S &= \int_{\mathcal{M}} d^3x\, L = \int_{\mathcal{M}} d^3x\, (L_X + L_\Psi + L_{\rm int} + L_{\rm pot} + L_{CS}), && (2.1)\\
L_X &= -\frac{1}{2}\langle D_\mu X^I, D^\mu X^I\rangle, && (2.2)\\
L_\Psi &= \frac{i}{2}\langle \bar\Psi, \Gamma^\mu D_\mu \Psi\rangle, && (2.3)\\
L_{\rm int} &= \frac{i}{4}\langle \bar\Psi, \Gamma_{IJ}[X^I, X^J, \Psi]\rangle, && (2.4)\\
L_{\rm pot} &= -\frac{1}{12}\langle [X^I, X^J, X^K], [X^I, X^J, X^K]\rangle, && (2.5)\\
L_{CS} &= \frac{1}{2} f^{ABCD} A_{AB}\wedge dA_{CD} + \frac{i}{3} f^{CDA}{}_G f^{EFGB} A_{AB}\wedge A_{CD}\wedge A_{EF}\,, && (2.6)
\end{aligned}$$

where the indices $\mu = 0,1,2$ specify the longitudinal directions of M2-branes, $I,J,K = 3,\ldots,10$ indicate the transverse directions, and the indices $A,B,C,\ldots$ denote components of 3-algebra generators. $\mathcal{M}$ is the worldvolume of M2-brane.

The covariant derivative is

$$(D_\mu \Phi(x))_A = \partial_\mu \Phi_A + f^{CDB}{}_A A_{\mu CD}(x)\Phi_B \tag{2.7}$$

for $\Phi = X^I, \Psi$. The 3-bracket for the 3-algebra in BLG model

$$[T^A, T^B, T^C] = i f^{ABC}{}_D T^D \tag{2.8}$$

must satisfy the fundamental identity and the invariant metric condition. Note that the notation is slightly different from the original BLG's one in order to make the field $A_{\mu AB}$ Hermite.

In [1], we made a systematic study of Lorentzian metric 3-algebra which contains $d+1$ pairs of Lorentzian metric generators (u_a, v^a) together with positive-definite generators e^i. We studied a special class of 3-algebra where the generators v^a are the center of 3-algebra, namely $[v^a, \star, \star] = 0$, and the generators u_a are not produced by the any 3-commutators, i.e. $f^{\star,\star,\star}{}_{u_a} = 0$. These requirements are necessary if we want to remove the ghost fields by using the Higgs mechanism in [6, 9]. A general feature for $d \geq 1$ is that the gauge fields (as well as all other fields, because of supersymmetry) become massive by absorbing two Higgs (ghost) scalar fields.

For finite dimensional 3-algebras, it is not obvious how to interpret these massive fields in the context of M/string theory. It was also found that BLG models based on known finite dimensional 3-algebras produce either products of the supersymmetric gauge theories [6–8] or abelian massive super Yang-Mills systems without interactions [1, 11].

For infinite dimensional case, it was found that there are varieties of possible 3-algebras and the BLG models associated with them in general have natural interpretation in M/string theory [1]. For example, while the number of particles becomes infinite, they

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are naturally interpreted as the Kaluza-Klein modes associated with the toroidal compactification. Also, the mass generated by ghosts can be identified with the Kaluza-Klein mass.

Here we pick up a 3-algebra which produces the worldvolume theory of Dp-brane ($p = d + 2$):[4]

$$[u_0, u_a, u_b] = 0\,, \tag{2.9}$$
$$[u_0, u_a, T^i_{\vec{m}}] = m_a T^i_{\vec{m}}\,, \tag{2.10}$$
$$[u_0, T^i_{\vec{m}}, T^j_{\vec{n}}] = m_a v^a \delta_{\vec{m}+\vec{n}} \delta^{ij} + i f^{ij}{}_k T^k_{\vec{m}+\vec{n}}\,, \tag{2.11}$$
$$[T^i_{\vec{l}}, T^j_{\vec{m}}, T^k_{\vec{n}}] = -i f^{ijk} \delta_{\vec{l}+\vec{m}+\vec{n}} v^0\,. \tag{2.12}$$

where $a, b = 1, \dots, d$, $\vec{l}, \vec{m}, \vec{n} \in \mathbf{Z}^d$ and f^{ijk} ($i, j, k = 1, \dots, \dim \mathbf{g}$) is a structure constant of an arbitrary Lie algebra $\mathbf{g}$ which satisfies Jacobi identity. Other 3-commutators are defined to be zero. The 3-algebra satisfies the fundamental identity. We note that v^A ($A = 0, 1, \dots, d$) are the center of the 3-algebra and u_A do not appear in the output of 3-commutators. This is an essential property of Lorentzian metric 3-algebra to make ghosts disappear after the Higgs mechanism. The nonvanishing part of the metric is given as

$$\langle u_A, v^B \rangle = \delta^B_A, \qquad \langle T^i_{\vec{m}}, T^j_{\vec{n}} \rangle = \delta^{ij} \delta_{\vec{m}+\vec{n}}\,. \tag{2.13}$$

We note that this 3-algebra can be regarded as original Lorentzian metric 3-algebra (1.1) where Lie algebra is replaced by

$$[u_a, u_b] = 0, \qquad [u_a, T^i_{\vec{m}}] = m_a T^i_{\vec{m}},$$
$$[T^i_{\vec{m}}, T^j_{\vec{n}}] = m_a v^a \delta_{\vec{m}+\vec{n}} \delta^{ij} + i f^{ij}{}_k T^k_{\vec{m}+\vec{n}}\,. \tag{2.14}$$

For $d = 1$, this is the standard Kac-Moody algebra with degree operator u and the central charge v and above algebra is its higher loop generalization. Since the original L-BLG model reduces to super Yang-Mills, one might guess that BLG model based on the 3-algebra (2.9)–(2.12) should be equivalent to super Yang-Mills whose gauge group is the loop algebra (2.14).[5] It turns out that this is not the case. As we explain below, BLG Lagrangian contains extra topological terms which can not be reproduced from Yang-Mills action.

2.2 Component expansion

In the remainder of this section, we will derive the BLG action for this 3-algebra. This was already presented in [1] but the computation is limited to the simplest choice of parameters and the dependence on the moduli parameter was not clarified. In particular, we will obtain some "topological" terms such as $\theta \int F\tilde{F}$ for D3-brane which could not show up for the simplest choice of the background. Furthermore, in order to obtain this θ term, we must carefully deal with the total derivative terms which is neglected in [1].

[4] In [1], more general 3-algebra is considered with the anti-symmetric tensor C_{ab}, i.e. $[u_0, u_a, u_b] = C_{ab} T^0_{\vec{0}}$ instead of eq. (2.9). This tensor is related with the noncommutativity parameter on Dp-brane. In this paper, we omit this factor for the simplicity of the argument.

[5] We note that the super Yang-Mills system with loop algebra symmetry is given in section 5.1 of [1].

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For the 3-algebra (2.9)–(2.12), we expand various fields as

$$X^I = X^I_{(i\vec{m})} T^i_{\vec{m}} + X^{IA} u_A + \underline{X}^I_A v^A \tag{2.15}$$

$$\Psi = \Psi_{(i\vec{m})} T^i_{\vec{m}} + \Psi^A u_A + \underline{\Psi}_A v^A \tag{2.16}$$

$$\begin{aligned} A_\mu = &\; A_{\mu(i\vec{m})(j\vec{n})} T^i_{\vec{m}} \wedge T^j_{\vec{n}} + \frac{1}{2} A_{\mu(i\vec{m})} u_0 \wedge T^i_{\vec{m}} + \frac{1}{2} A^a_{\mu(i\vec{m})} u_a \wedge T^i_{\vec{m}} \\ & + \frac{1}{2} A^a_\mu \, u_0 \wedge u_a + A^{ab}_\mu \, u_a \wedge u_b + (\text{terms including } v^A) \, . \end{aligned} \tag{2.17}$$

Now we will rewrite the BLG action (2.1) as an action for Dp-branes ($p = d + 2$). More precisely, if we denote the original membrane worldvolume as $\mathcal{M}$, the worldvolume of Dp-brane is given by a flat T^d bundle over $\mathcal{M}$. The index $\vec{m} \in \mathbf{Z}^d$ which appears in some components represents the Kaluza-Klein momentum along the T^d.

In this geometrical set-up, each bosonic components plays the following roles:

- $X^I_{(i\vec{m})}$: These are splitted into three groups. Some are the collective coordinates which describe the embedding into the transverse directions, others are the gauge fields on the worldvolume, and the other is the degree of freedom which can be absorbed when M-direction disappears. The concrete expression is eq. (2.55).
- X^{IA} : Higgs fields whose VEV's determine either the moduli of T^d or the compactification radius in M-direction.
- $A_{\mu(i\vec{m})}$: gauge fields along the membrane worldvolume $\mathcal{M}$.
- A^a_μ : a connection which describes the fiber bundle $T^d \to \mathcal{M}$. The equation of motion implies that it is always flat $\partial_{[\mu} A^a_{\nu]} = 0$.

The other bosonic components become Lagrange multiplier or do not show up in the action at all. In the following, we set $A^a_\mu = A^{ab}_\mu = 0$ for simplicity.

2.3 Solving the ghost sector

The components of ghost fields $\underline{X}$ and $\underline{\Psi}$ appear in the action only through the following terms:

$$L_{gh} = -(D_\mu X^I)_{u_A} (D_\mu X^I)_{v^A} + \frac{i}{2} \left(\bar{\Psi}_{u_A} \Gamma^\mu D_\mu \Psi_{v^A} + \bar{\Psi}_{v^A} \Gamma^\mu D_\mu \Psi_{u_A} \right) \tag{2.18}$$

where

$$\begin{aligned} (D_\mu X^I)_{u_A} &= \partial_\mu X^{IA} \, , \\ (D_\mu X^I)_{v^0} &= \partial_\mu \underline{X}^I_0 + i m_a (A^a_{\mu(i\vec{m})} X^I_{(i,-\vec{m})} + A_{\mu(i\vec{m})(i,-\vec{m})} X^{Ia}) \\ &\quad - f^{ijk} A_{\mu(i\vec{m})(j\vec{m})} X^I_{(k,-\vec{m}-\vec{n})} \, , \\ (D_\mu X^I)_{v^a} &= \partial_\mu \underline{X}^I_a - i m_a (A_{\mu(i\vec{m})} X^I_{(i,-\vec{m})} + A_{\mu(i\vec{m})(i,-\vec{m})} X^{I0}) \, , \end{aligned} \tag{2.19}$$

and similar for Ψ. The variation of $\underline{X}^I_A$ and $\underline{\Psi}_A$ always give the *free* equations of motion for X^{IA} and Ψ^A, namely

$$\partial^\mu \partial_\mu X^{IA} = 0 \, , \qquad \Gamma^\mu \partial_\mu \Psi^A = 0 \, . \tag{2.20}$$

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By introducing extra gauge fields $C^I_{\mu A}$ and χ_A through [15, 16]

$$L_{\rm new} = C^I_{\mu A}\partial_\mu X^{IA} - \chi_A \bar{\Psi}^A\,, \tag{2.21}$$

one may modify the equations of motion for X^{IA} and Ψ^A to

$$\partial_\mu X^{IA} = 0\,, \qquad \Psi^A = 0\,, \tag{2.22}$$

and absorb the ghosts $\underline{X}^I_A$ and $\underline{\Psi}_A$ by gauge fixing. This is how the ghost fields can be removed in [6–8].

The equations of motion for X^{IA} (2.22) imply that they are constant vectors in $\mathbf{R}^8$. We fix these constants as

$$\vec{X}^A = \vec{\lambda}^A \in \mathbf{R}^{d+1} \subset \mathbf{R}^8\,. \tag{2.23}$$

In [6–8], there is only one $\vec{\lambda} = \vec{\lambda}^0$ which specifies the M-direction compactification radius. This time, we have extra VEV's $\vec{\lambda}^a$ which give the moduli of the toroidal compactification T^d.

In the following, we prepare some notations for the later discussion. We write the dual basis to $\vec{\lambda}^A$ as $\vec{\pi}_A$, which satisfy

$$\vec{\lambda}^A \cdot \vec{\pi}_B = \delta^A_B\,. \tag{2.24}$$

We introduce a projector into the subspace of $\mathbf{R}^8$ which is orthogonal to all $\vec{\lambda}^A$ as

$$P^{IJ} = \delta^{IJ} - \sum_A \lambda^{IA}\pi^J_A\,, \tag{2.25}$$

which satisfies $P^2 = P$. We define 'metric' as

$$G^{AB} = \vec{\lambda}^A \cdot \vec{\lambda}^B\,, \tag{2.26}$$

where λ^{IA} play the role of vierbein. Using this metric, $\vec{\pi}_0$ can be written as

$$\vec{\pi}_0 = \frac{1}{G^{00}}\,\vec{\lambda}^0 - \frac{G^{0a}}{G^{00}}\,\vec{\pi}_a\,, \tag{2.27}$$

and from now we use $\{\vec{\lambda}^0, \vec{\pi}_a\}$ as the basis of $\mathbf{R}^{d+1}$ spanned by $\vec{\lambda}^A$. Note that $\vec{\lambda}^0 \perp \vec{\pi}_a$ for all a. Our claim that the $\mathbf{R}^{d+1}$ is compactified on T^{d+1} will be deduced from the Kaluza-Klein mass which is generated by the Higgs mechanism. This will be demonstrated below.

Comments on Higgs potential. Since $\vec{X}^A$ plays the role of Higgs fields, it is natural to wonder if one may introduce a potential for them and fix the value of VEV's. This seems to be physically relevant since they are related to the moduli of torus. One naive guess is to add a potential $-V(\vec{X}^A)$ to the action. Since the SUSY and gauge transformations of $\vec{X}^A$ are trivial, this potential breaks neither SUSY nor gauge symmetry. However, the kinetic term is given in the mixed form $\partial\vec{X}^A\partial\underline{\vec{X}}_A$, the potential does not fix $\vec{X}^A$ but physically irrelevant $\underline{\vec{X}}_A$.

2.4 Derivation of Dp-brane action

We finally rewrite the BLG action (2.1) in terms of 3-algebra components and by putting VEV's to ghost fields X^{IA} and Ψ^A.

Kinetic terms for X^I and Ψ. The covariant derivative becomes, after the assignment of VEV's to ghosts,

$$(D_\mu X^I)_{(i\vec{m})} = (\hat{D}_\mu X^I)_{(i\vec{m})} + A'_{\mu(i\vec{m})}\lambda^{I0} - im_a A_{\mu(i\vec{m})}\lambda^{Ia} \tag{2.28}$$

where

$$(\hat{D}_\mu X^I)_{(i\vec{m})} = \partial_\mu X^I_{(i\vec{m})} + f^{jk}{}_i A_{\mu(k\vec{n})} X^I_{(j,\vec{m}-\vec{n})}\,, \tag{2.29}$$

$$A'_{\mu(i\vec{m})} = -im_a A^a_{\mu(i\vec{m})} + f^{jk}{}_i A_{\mu(j,\vec{m}-\vec{n})(k\vec{n})}\,. \tag{2.30}$$

We decompose this formula into the components into the orthogonal spaces $\mathbf{R}^{7-d}$ and $\mathbf{R}^{d+1}$ by using the projector P^{IJ} as

$$(D_\mu X^I)_{(i\vec{m})} = P^{IJ}(\hat{D}_\mu X^J)_{(i\vec{m})} + \sum_A \lambda^{IA}(F_{\mu A})_{(i\vec{m})} \tag{2.31}$$

where

$$\begin{aligned}(F_{\mu 0})_{(i\vec{m})} &= \vec{\pi}_0\cdot(\hat{D}_\mu \vec{X})_{(i\vec{m})} + A'_{\mu(i,\vec{m})}\\ &= \frac{1}{G^{00}}\hat{D}_\mu(\vec{\lambda}^0\cdot\vec{X})_{(i\vec{m})} - \frac{G^{0a}}{G^{00}}\hat{D}_\mu(\vec{\pi}_a\cdot\vec{X})_{(i\vec{m})} + A'_{\mu(i\vec{m})}\,,\end{aligned} \tag{2.32}$$

$$(F_{\mu a})_{(i\vec{m})} = \hat{D}_\mu(\vec{\pi}_a\cdot\vec{X})_{(i\vec{m})} - im_a A_{\mu(i\vec{m})}\,. \tag{2.33}$$

We will rewrite $\vec{\pi}_a\cdot\vec{X}$ as A_a below, since they play the role of gauge fields along the fiber T^d as we mentioned. $F_{\mu a}$ will be regarded as the field strength with one leg in $\mathcal{M}$ and the other in T^d. $F_{\mu 0}$ seems to be the field strength in a similar sense with one leg in M-direction. However, the gauge field $A'_{\mu(i\vec{m})}$ is an auxiliary field as we see below, and after it is integrated out, $F_{\mu 0}$ will completely disappear from the action. In this sense, $F_{\mu 0}$ do not have any geometrical meaning. We suspect, however, that it may give a hint to keep the trace of the compactification of M-theory to type IIA superstring theory.

Finally, using eq. (2.31), the kinetic term for X^I becomes

$$L_X = -\frac{1}{2}\hat{D}_\mu X^I_{(i\vec{m})} P^{IJ}\hat{D}_\mu X^J_{(i,-\vec{m})} - \frac{1}{2}G^{AB}F_{\mu A(i\vec{m})}F_{\mu B(i,-\vec{m})}\,. \tag{2.34}$$

Similarly, the kinetic term for Ψ becomes

$$L_\Psi = \frac{i}{2}\bar{\Psi}_{(i\vec{m})}\Gamma^\mu\hat{D}_\mu\Psi_{(i,-\vec{m})}\,. \tag{2.35}$$

Chern-Simons term and integration of A'. The Chern-Simons term is written as

$$\begin{aligned} L_{CS} &= \frac{1}{2}\left(A'_{(i\vec{m})}\wedge dA_{(i,-\vec{m})} + A_{(i,-\vec{m})}\wedge dA'_{(i\vec{m})}\right) \\ &\quad -if^{ijk}A'_{(i\vec{m})}\wedge A_{(j\vec{n})}\wedge A_{(k,-\vec{m}-\vec{n})}\,, \end{aligned} \tag{2.36}$$

or, up to the total derivative terms,

$$L_{CS} = \frac{1}{2}A'_{(i\vec{m})}\wedge F_{(i,-\vec{m})} + (\text{total derivative})\,, \tag{2.37}$$

where

$$F_{\mu\nu(i\vec{m})} = \partial_\mu A_{\nu(i\vec{m})} - \partial_\nu A_{\mu(i\vec{m})} + f^{jk}{}_i A_{\mu(j\vec{n})}A_{\nu(k,\vec{m}-\vec{n})}\,. \tag{2.38}$$

Since the gauge field A' shows up only in L_{CS} and L_X, one may algebraically integrate over it. Variation of A' gives the equation of motion

$$\begin{aligned} A'_{\mu(i,\vec{m})} &= -\frac{1}{G^{00}}\hat{D}_\mu(\vec{\lambda}^0\cdot\vec{X})_{(i\vec{m})} + \frac{G^{0a}}{G^{00}}\hat{D}_\mu A_{a(i\vec{m})} - \frac{G^{0a}}{G^{00}}(F_{\mu a})_{(i\vec{m})} \\ &\quad -\frac{1}{2G^{00}}\epsilon_{\mu\nu\lambda}(F_{\nu\lambda})_{(i\vec{m})}\,, \end{aligned} \tag{2.39}$$

where $A_a := \vec{\pi}_a\cdot\vec{X}$. By putting back this value to the original action (2.36),

$$\begin{aligned} L_X + L_{CS} &= -\frac{1}{2}\hat{D}_\mu X^I P^{IJ}\hat{D}_\mu X^J - \frac{1}{4G^{00}}(F_{\nu\lambda})^2 - \frac{1}{2}\tilde{G}^{ab}F_{\mu a}F_{\mu b} \\ &\quad -\frac{G^{0a}}{2G^{00}}\epsilon^{\mu\nu\lambda}F_{\mu a}F_{\nu\lambda} + L_{td}\,, \end{aligned} \tag{2.40}$$

where

$$\tilde{G}^{ab} := G^{ab} - \frac{G^{a0}G^{b0}}{G^{00}}\,, \tag{2.41}$$

$$L_{td} = -\frac{1}{2G^{00}}\epsilon_{\mu\nu\lambda}\partial_\mu\left[\left(-i\hat{D}_\nu(\vec{\lambda}^0\cdot\vec{X}) + \frac{1}{2}\epsilon_{\nu\rho\sigma}F_{\rho\sigma}\right)A_\lambda\right]\,. \tag{2.42}$$

Here we omit the indices $(i\vec{m})$ for simplicity. Note that the redefinition of the metric $G^{ab}\to\tilde{G}^{ab}$ is very similar to that of T-duality transformation in M-direction. The term L_{td} is total derivative which does not vanish in the limit $G^{0a}\to 0$. Since we know that the total derivative terms do not play any role for the case $G^{0a} = \vec{\lambda}^0\cdot\vec{\lambda}^a = 0$, we will neglect them in the following. In a sense, this is equivalent to redefine the BLG action,

$$S_{BLG} = \int d^3x\,(L_{BLG} - L_{td})\,, \tag{2.43}$$

where L_{BLG} is the original BLG Lagrangian. On the other hand, while the fourth term in eq. (2.40) is also total derivative, we must *not* neglect it. This is because this term is proportional to G^{0a} and becomes essential to understand the U-duality. For $d = 1$ case, it becomes the θ term of the super Yang-Mills action and it should be involved in the S-duality transformation in the complex coupling constant $\tau = C_0 + ie^{-\phi}$. We note that this is the term which does not show up if we analyze the Yang-Mills system with loop algebra symmetry (2.14).

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Kaluza-Klein mass by Higgs mechanism. At this point, it is easy to understand how compactification occurs after the Higgs mechanism. Note that in the definition of $F_{\mu a}$ (2.33), we have a factor with m_a in front of $A_{\mu(i\vec{m})}$. In the language of D2-brane worldvolume, it gives rise to the mass term

$$-\frac{1}{2}g^{ab}m_a m_b A_{\mu(i\vec{m})}A^{\mu}_{(i,-\vec{m})}\,, \qquad \text{where} \quad g^{ab} := G^{00}\tilde{G}^{ab}, \tag{2.44}$$

for $A_{\mu(i\vec{m})}$. We will also see that exactly the same mass term exists for all fields with index $\vec{m}$. It is natural to regard these terms as the Kaluza-Klein mass terms for the compactification on a torus T^d.

In order to be more explicit, we will use the T-dual picture [19] in the following. We identify the various fields with index $\vec{m}$ with the higher $3+d$ dimensional fields by the identification

$$\Phi_{\vec{m}}(x) \to \tilde{\Phi}(x,y) := \sum_{\vec{m}} \Phi_{\vec{m}}(x)e^{i\vec{m}\vec{y}} \tag{2.45}$$

where $y^a \in [0, 2\pi]$ $(a = 1, \dots, d)$ are coordinates of T^d. $F_{\mu a}$ can be identified with the field strength by

$$(\tilde{F}_{\mu a})_i = \hat{D}_\mu \tilde{A}_{ai} - \frac{\partial}{\partial y^a}\tilde{A}_{\mu i} \tag{2.46}$$

where $\tilde{A}_{ai}(x,y) := \vec{\pi}_a \cdot \vec{\tilde{X}}_i(x,y)$. The kinetic terms of gauge fields in eq. (2.40) imply that we have a metric in $\vec{y}$ direction as

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + g_{ab}\, dy^a dy^b\,, \qquad \text{where} \quad g_{ab} := (g^{ab})^{-1}\,. \tag{2.47}$$

When $\vec{\lambda}^A$ are all orthogonal, one may absorb the metric g_{ab} in the rescaling of y^a as $y'^a = (|\vec{\lambda}^0||\vec{\lambda}^a|)^{-1}y^a$. Since y^a has the radius 1, y'^a has the radius $1/|\vec{\lambda}^0||\vec{\lambda}^a|$. This is consistent with our previous analysis [1]. In this scaling $y^a \to y'^a$, the kinetic terms for gauge fields in eq. (2.40) become

$$-\frac{1}{4G^{00}}\left[(F_{\nu\lambda})^2 + 2(F_{\mu a})^2\right]\,, \tag{2.48}$$

which is also consistent with our previous study for $d = 1$.

We note that the use of Kac-Moody algebra as the symmetry of the Kaluza-Klein mode is not new. See, for example, [22]. Here the novelty is to use the Higgs mechanism to obtain the Kaluza-Klein mass.

Worldvolume is a flat fiber bundle. So far, since we put $A^a_\mu = 0$ for the simplicity of the argument, the worldvolume of Dp-brane is the product space $\mathcal{M} \times T^d$. In order to see the geometrical role of A^a_μ, let us keep it nonvanishing for a moment. The covariant derivative (2.28) get an extra term, $m_a A^a_\mu(x) X^I_{(i\vec{m})}$, which becomes on $\mathcal{M} \times T^d$,

$$iA^a_\mu(x)\frac{\partial}{\partial y^a}\tilde{X}^I_i(x,y)\,. \tag{2.49}$$

A^a_μ turns out to be the gauge field for the gauge transformation from those of BLG:

$$\delta \tilde{X}^I_i(x,y) = i\gamma^a(x)\frac{\partial}{\partial y^a}\tilde{X}^I_i(x,y)\,. \tag{2.50}$$

The existence of the gauge coupling implies that the worldvolume is not the direct product $\mathcal{M}\times T^d$ but a fiber bundle Y:

$$\begin{array}{ccc} T^d & \longrightarrow & Y \\ & & \downarrow \\ & & \mathcal{M} \end{array}$$

where T^d act as the translation of y^a.

The kinetic term for the connection comes from the Chern-Simons term:

$$L_{\text{fiber}} = \epsilon^{\mu\nu\lambda}C_{\mu a}\partial_\nu A^a_\lambda\,, \qquad C_{\mu a} := \sum_{\vec{n}} n_a A_{\mu(i\vec{n})(i,-\vec{n})}\,. \tag{2.51}$$

Since $C_{\mu a}$ does not appear in other place in the action, its variation gives,

$$\partial_{[\mu}A^a_{\nu]} = 0\,. \tag{2.52}$$

Therefore Y must be a flat bundle as long as we start from BLG model.

There seems to be various possibilities to relax this constraint to the curved background. One naive guess is to replace L_{fiber} to

$$L'_{\text{fiber}} = \epsilon^{\mu\nu\lambda}C_{\mu a}\left(\partial_\nu A^a_\lambda - \frac{1}{2}F^{a(0)}_{\nu\lambda}\right)\,, \tag{2.53}$$

for an appropriate classical background $F^{a(0)}_{\nu\lambda}$.

Interaction terms. The compactification picture works as well in the interaction terms. For the fermion interaction term L_{int}, we use

$$[X^{[I}, X^{J]}, \Psi]_{(i,-\vec{m})} = -m_a\lambda^{[I0}\lambda^{J]a}\Psi_{(i,-\vec{m})} + if^{jk}{}_i\lambda^{[I0}X^{J]}_{(j\vec{n})}\Psi_{(k,-\vec{m}-\vec{n})} \tag{2.54}$$

and from eq. (2.27),

$$\begin{aligned} X^I &= P^{IJ}X^J + \lambda^{IA}(\vec{\pi}_A\cdot\vec{X}) \\ &= P^{IJ}X^J + \frac{1}{G^{00}}\lambda^{I0}(\vec{\lambda}^0\cdot\vec{X}) + \left(-\frac{G^{0a}}{G^{00}}\lambda^{I0} + \lambda^{Ia}\right)A_a\,. \end{aligned} \tag{2.55}$$

Then L_{int} can be written as

$$\begin{aligned} L_{\text{int}} &= \frac{i}{4}\bar{\Psi}_{(i\vec{m})}(\Gamma_{IJ}\lambda^{I0}\lambda^{Ja})\left(-m_a\Psi_{(i,-\vec{m})} + if^{jk}{}_iA_{a(j\vec{n})}\Psi_{(k,-\vec{m}-\vec{n})}\right) \\ &\quad + \frac{i}{4}\bar{\Psi}_{(i\vec{m})}(\Gamma_{IJ}\lambda^{I0})\left(if^{jk}{}_iP^{JK}X^K_{(j\vec{n})}\Psi_{(k,-\vec{m}-\vec{n})}\right) \\ &= \int\frac{d^dy}{(2\pi)^d}\sqrt{g}\left(\frac{i}{2}\bar{\tilde{\Psi}}\Gamma^a\hat{D}_a\tilde{\Psi} + \frac{i\sqrt{G^{00}}}{2}\bar{\tilde{\Psi}}\Gamma_I[P^{IJ}\tilde{X}^J, \tilde{\Psi}]\right)\,, \end{aligned} \tag{2.56}$$

where $g = \det\, g^{ab}$, $\hat{D}_a\tilde{\Psi} := \partial_a\tilde{\Psi} - i[\tilde{A}_a, \tilde{\Psi}]$ and

$$\Gamma^a := \frac{i}{2}\Gamma_{IJ}\lambda^{I0}\lambda^{Ja}\,, \qquad \Gamma_J := \frac{1}{2\sqrt{G^{00}}}\Gamma_{IJ}\lambda^{I0}\,, \tag{2.57}$$

which satisfy $\{\Gamma^a, \Gamma^b\} = g^{ab}$ and $\{\Gamma_I, \Gamma_J\} = \delta_{IJ}$.

On the other hand, the potential term for the boson $L_{\rm pot}$ is the square of a 3-commutator:

$$[X^I, X^J, X^K]_{(i,\vec{m})} = m_a\lambda^{[I0}\lambda^{Ja}X^{K]}_{(i,\vec{m})} + if^{jk}{}_i\lambda^{[I0}X^J_{(j,\vec{n})}X^{K]}_{(k,\vec{m}-\vec{n})}. \tag{2.58}$$

where the indices I, J, K are antisymmetrized. The square of the first term gives

$$\left(m_a\lambda^{[I0}\lambda^{Ja}X^{K]}_{(i,\vec{m})}\right)^2 = 6g^{ab}m_am_bX^I_{\vec{m}}P^{IJ}_{\vec{m}}X^J_{-\vec{m}}\,, \tag{2.59}$$

where

$$\begin{aligned} P^{IJ}_{\vec{m}} &:= \delta^{IJ} - \frac{|\vec{\lambda}^0|^2\lambda^I_{\vec{m}}\lambda^J_{\vec{m}} + |\lambda_{\vec{m}}|^2\vec{\lambda}^{I0}\vec{\lambda}^{J0} - (\vec{\lambda}^0\cdot\vec{\lambda}_{\vec{m}})(\lambda^{I0}\lambda^J_{\vec{m}} + \lambda^{J0}\lambda^I_{\vec{m}})}{|\vec{\lambda}^0|^2|\vec{\lambda}_{\vec{m}}|^2 - (\vec{\lambda}^0\cdot\vec{\lambda}_{\vec{m}})^2}\,, \\ \vec{\lambda}_{\vec{m}} &:= m_a\vec{\lambda}^a\,, \end{aligned} \tag{2.60}$$

which satisfy

$$P^{IJ}_{\vec{m}}\lambda^{J0} = P^{IJ}_{\vec{m}}\lambda^J_{\vec{m}} = 0\,, \qquad P^2_{\vec{m}} = P_{\vec{m}}\,. \tag{2.61}$$

The mixed term

$$\lambda^{[I0}\lambda^J_{\vec{m}}X^{K]}_{(i\vec{m})}\cdot f^{jk}{}_i\lambda^{[I0}X^J_{(j,\vec{n})}X^{K]}_{(k,-\vec{m}-\vec{n})} \tag{2.62}$$

vanishes and does not contribute to the action. The commutator part is

$$(if^{jk}{}_i\lambda^{[I0}X^J_{(j,\vec{n})}X^{K]}_{(k,\vec{m}-\vec{n})})^2 = 3\left(G^{00}\langle[X^J, X^K]^2\rangle - 2\langle[(\vec{\lambda}^0\cdot\vec{X}), X^I]^2\rangle\right) \tag{2.63}$$

which is identical to the similar term in [6] and it produces the standard commutator terms. Using eq. (2.55), these terms can be summarized in the following compact form:

$$\begin{aligned} L_{\rm pot} = \int \frac{d^dy}{(2\pi)^d}\sqrt{g}\,\Bigg(&-\frac{1}{2}g^{ab}\hat{D}_a\tilde{X}^IP^{IJ}\hat{D}_b\tilde{X}^J - \frac{1}{4G^{00}}g^{ac}g^{bd}\tilde{F}_{ab}\tilde{F}_{cd} \\ &-\frac{G^{00}}{4}[P^{IK}\tilde{X}^K, P^{JL}\tilde{X}^L]^2\Bigg)\,, \end{aligned} \tag{2.64}$$

where $\hat{D}_a\tilde{X}^I = \partial_a\tilde{X}^I - i[\tilde{A}_a, \tilde{X}^I]$ and $\tilde{F}_{ab} = \partial_a\tilde{A}_b - \partial_b\tilde{A}_a - i[\tilde{A}_a, \tilde{A}_b]$.

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2.5 Summary

By collecting all the results in previous subsections, the BLG action (2.1) becomes

$$L = L_A + L_{FF} + L_X + L_\Psi + L_{\rm pot} + L_{\rm int} + L_{td}\,, \tag{2.65}$$

$$L_A = -\frac{1}{4G^{00}}\int \frac{d^dy}{(2\pi)^d}\sqrt{g}\left(\tilde{F}_{\mu\nu}^2 + 2g^{ab}\tilde{F}_{\mu a}\tilde{F}_{\mu b} + g^{ac}g^{bd}\tilde{F}_{ab}\tilde{F}_{cd}\right)\,, \tag{2.66}$$

$$L_{FF} = -\frac{G^{0a}}{8G^{00}}\int \frac{d^dy}{(2\pi)^d}\sqrt{g}\left(4\epsilon^{\mu\nu\lambda}\tilde{F}_{\mu a}\tilde{F}_{\nu\lambda}\right)\,, \tag{2.67}$$

$$L_X = -\frac{1}{2}\int \frac{d^dy}{(2\pi)^d}\sqrt{g}\left(\hat{D}_\mu\tilde{X}^I P^{IJ}\hat{D}_\mu\tilde{X}^J + g^{ab}\hat{D}_a\tilde{X}^I P^{IJ}\hat{D}_b\tilde{X}^J\right)\,, \tag{2.68}$$

$$L_\Psi = \frac{i}{2}\int \frac{d^dy}{(2\pi)^d}\sqrt{g}\,\bar{\tilde{\Psi}}\left(\Gamma^\mu\hat{D}_\mu + \Gamma^a\hat{D}_a\right)\tilde{\Psi}\,, \tag{2.69}$$

$$L_{\rm pot} = -\frac{G^{00}}{4}\int \frac{d^dy}{(2\pi)^d}\sqrt{g}\,[P^{\rm IK}\tilde{X}^K, P^{JL}\tilde{X}^L]^2\,, \tag{2.70}$$

$$L_{\rm int} = \frac{i\sqrt{G^{00}}}{2}\int \frac{d^dy}{(2\pi)^d}\sqrt{g}\,\bar{\tilde{\Psi}}\Gamma_I[P^{IJ}\tilde{X}^J,\tilde{\Psi}]\,. \tag{2.71}$$

It is easy to see that this is the standard Dp-brane action ($p = d+2$) on $\mathcal{M}\times T^d$ with the metric (2.47). Interpretation and implications of this action are given in the next section.

3 Study of U-duality in L-BLG model

3.1 D3-branes case

For $d = 1$, if we write $\vec{\lambda}^0 = \vec{e}^{\,0}$, $\vec{\lambda}^1 = \tau_1\vec{e}^{\,0} + \tau_2\vec{e}^{\,1}$ (where $\vec{e}^{\,0}\cdot\vec{e}^{\,1} = 0$, $|\vec{e}^{\,0}| = |\vec{e}^{\,1}|$), the action for the gauge field is given as

$$\begin{aligned} L_A + L_{FF} &= -\frac{1}{4G^{00}}\int\frac{dy}{2\pi}\sqrt{g}\,F^2 - \frac{G^{01}}{8G^{00}}\int\frac{dy}{2\pi}\,F\widetilde{F} \\ &= -\frac{1}{8\pi}\int dy\left(\frac{\tau_1}{2}F\widetilde{F} + \tau_2 F^2\right) \end{aligned} \tag{3.1}$$

where now $g = g^{11}$ and

$$\begin{aligned} F^2 &= \tilde{F}_{\mu\nu}^2 + 2g^{11}\tilde{F}_{\mu 1}\tilde{F}_{\mu 1}\,, \\ F\widetilde{F} &= (4\sqrt{g^{11}}\,\epsilon^{\mu\nu\lambda})\tilde{F}_{\mu 1}\tilde{F}_{\nu\lambda}\,. \end{aligned} \tag{3.2}$$

This shows that the action (2.65) in this case is the standard D3-brane action with the θ term.

Under the SL$(2,\mathbf{Z})$ transformation

$$\begin{pmatrix}\vec{\lambda}^1\\ \vec{\lambda}^0\end{pmatrix} \to \begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}\vec{\lambda}^1\\ \vec{\lambda}^0\end{pmatrix}\,, \tag{3.3}$$

the moduli parameter τ is transformed as,

$$\tau \to \frac{a\tau + b}{c\tau + d}\,. \tag{3.4}$$

For $b = -c = 1$, $a = d = 0$, it gives rise to the standard S-duality transformation $\tau \to -1/\tau$. On the other hand, $a = d = 1$, $b = n$ and $c = 0$ gives the translation $\tau \to \tau + n$.

We do not claim that we have proven S-duality symmetry from our model. At the level of 3-algebra (2.9)–(2.12), there is obvious asymmetry between u_0, v^0 and u_1, v^1. Nevertheless, it is illuminating that the S-duality symmetry can be interpreted in so simple way.

On the other hand, the translation symmetry reduces to the automorphism of the 3-algebra (2.9),

$$\begin{aligned} u_0 &\to u_0 - n u_1\,, & u_1 &\to u_1\,,\\ v^0 &\to v^0\,, & v^1 &\to v^1 + n v^0\,. \end{aligned} \tag{3.5}$$

It is easy to see that the transformation changes neither 3-algebra nor their metric. It induces the redefinition the ghost fields as,

$$X^I = X^I_{u_0} u_0 + X^I_{u_1} u_1 + \cdots = X^I_{u_0}(u_0 - n u_1) + (X^I_{u_1} + n X^I_{u_0})\, u_1 + \cdots\,. \tag{3.6}$$

It implies the transformation $\vec{\lambda}^0 \to \vec{\lambda}^0\,,\ \vec{\lambda}^1 \to \vec{\lambda}^1 + n\vec{\lambda}^0\,$. Of course, at the classical level, there is no reason that the parameter n must be quantized. It is interesting anyway that part of the duality transformation comes from the automorphism of 3-algebra.

The T-duality transformation $\mathbf{Z}_2$ which interchanges D3- and D2-branes comes from the different identification of component fields. Namely, we have constructed 4-dimensional field $\tilde{X}^I(x,y)$ from the component fields $X^I_{(i\vec{m})}(x)$ by Fourier series (2.45). One may instead interpret $X^I_{(i\vec{m})}(x)$ as the 3-dimensional field and interpret $\vec{m}$ index as describing open string mode which interpolate mirror images of a point in $T^1 = \mathbf{R}/\mathbf{Z}$. This is the standard T-duality argument [19].

The relation between the coupling constant and the radius in T-duality transformation is given as follows. Let us assume for a moment that $\vec{\lambda}^0 \perp \vec{\lambda}^1$ for simplicity. It is well known [9] that putting a VEV $\vec{X}_{u_0} = \vec{\lambda}^0$ means the compactification of M-direction with the radius

$$R_0 = |\vec{\lambda}^0|\, l_p^{3/2}\,, \tag{3.7}$$

where l_p is 11-dimensional Planck length. From the symmetry of $X_{u_0} \leftrightarrow X_{u_1}$, putting a VEV $\vec{X}_{u_1} = \vec{\lambda}^1$ must imply the compactification of another direction with the similar radius $\tilde{R}_1 = |\vec{\lambda}^1|\, l_p^{3/2}$ before taking T-duality along $\vec{\lambda}^1$. At this point, we have D2-brane worldvolume theory with string coupling

$$g_s = g_{YM}^2\, l_s = |\vec{\lambda}^0|^2\, l_s\,. \tag{3.8}$$

where l_s is the string length, satisfying $l_p^3 = g_s l_s^3$. In section 2, we obtain D3-brane since we compactify the $\vec{\lambda}^1$ direction with radius $\tilde{R}_1$ and simultaneously take T-duality for the same direction. Thus the D3-brane is compactified on S^1 with the radius

$$R_1 = \frac{l_s^2}{\tilde{R}_1} = \frac{l_s^2}{|\vec{\lambda}^1|\sqrt{|\vec{\lambda}^0|^2\, l_s^4}} = \frac{1}{|\vec{\lambda}^0||\vec{\lambda}^1|}\,, \tag{3.9}$$

and the string coupling for D3-brane worldvolume theory is

$$g_s' = g_s \frac{l_s}{\tilde{R}_1} = \frac{|\vec{\lambda}^0|}{|\vec{\lambda}^1|} . \tag{3.10}$$

This result is consistent with our result [1], as we also discussed in section 2.

To summarize, the U-duality transformation for $d = 1$ case is

$$\mathrm{SL}(2, \mathbf{Z}) \bowtie \mathbf{Z}_2 , \tag{3.11}$$

where the first factor is described by the rotation of Higgs VEV's and the second factor is described by the different representation as the field theory.

3.2 U-duality for $d > 1$

We consider M-theory compactified on T^{d+1} (where $d = p - 2$). This theory has U-duality group $E_{d+1}(\mathbf{Z})$ and scalars taking values in E_{d+1}/H_{d+1} where H_{d+1} is the maximal compact subgroup of E_{d+1}. See, for example, [20] for detail. We call the space of these scalars 'parameter space' in the following.

In this subsection, we compare the parameters obtained from L-BLG model with those in the parameter space. We can extract various parameters on Dp-brane from the action obtained in section 2.5 which are all determined by the Higgs VEV's $\vec{\lambda}^A$. The first one is the Yang-Mills coupling:

$$g_{YM}^2 = \frac{(2\pi)^d G^{00}}{\sqrt{g}} , \qquad g := \det\, g^{ab} . \tag{3.12}$$

Secondly, the metric

$$g^{ab} = G^{00} G^{ab} - G^{0a} G^{0b} \tag{3.13}$$

gives the moduli of the torus T^d. Finally, L_{FF} gives a generalization of θ term for $d = 1$ case. Since the θ term may be regarded as the axion coupling, a natural generalization for general d is the R-R field $C_{(d-1)}$, which appears in the Dp-brane Lagrangian of string theory like as $C_{(d-1)} \wedge F \wedge F$. Such term was discussed in the literature, for example, in [20].

In our set-up in section 2, the existence of such coupling $C \wedge F \wedge F$ can be understood as follows. There the compactification of the M-direction was determined by $\vec{\lambda}^0$ and we took T-duality on T^d specified by $\{\vec{\lambda}^a\} = \{\vec{\lambda}^1, \ldots, \vec{\lambda}^d\}$. If $G^{0a} = \vec{\lambda}^0 \cdot \vec{\lambda}^a \neq 0$, we obtain the nonzero $C_{(0)}$ field, after the compactification of M-direction and the T-duality transformation along only y^a. After taking T-duality in the remaining $d - 1$ directions on T^d too, we obtain the nonzero $C_{(d-1)}$ field whose nonvanishing component is $C_{1\cdots\hat{a}\cdots d}$, where the index with $\hat{}$ should be erased. This compontent of R-R field must interact with gauge fields on D-brane as $\epsilon^{\mu\nu\lambda 1\cdots d} C_{1\cdots\hat{a}\cdots d} F_{\mu\nu} F_{\lambda a}$. In our action (2.65), L_{FF} describes this coupling. It determines the components of $C_{(d-1)}$ as

$$C_{\hat{a}} := C_{1\cdots\hat{a}\cdots d} = \frac{1}{4(2\pi)^d (d-1)!} \frac{G^{0a}}{G^{00}} \frac{\sqrt{g}}{\sqrt{g^{aa}}} , \tag{3.14}$$

where no sum is taken on a.

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The number of parameters thus obtained is $1+\frac{d(d+1)}{2}+d=\frac{(d+1)(d+2)}{2}$ which coincides with the number of metric $G^{AB}=\vec{\lambda}^A\cdot\vec{\lambda}^B$. As is $d=1$ case, it is natural to guess the $\mathrm{SL}(d+1,\mathbf{Z})$ transformation

$$\vec{\lambda}'^A=\Lambda^A{}_B\,\vec{\lambda}^B\,,\qquad \Lambda^A{}_B\in\mathrm{SL}(d+1,\mathbf{Z})\,,\tag{3.15}$$

is related to the first factor of $\mathrm{SL}(d+1,\mathbf{Z})\bowtie O(d,d:\mathbf{Z})$ in U-duality transformation. In appendix A, we derive the transformation law of these parameters explicitly. They are less illuminative compared with $d=1$ case, however, since these parameters depends on G^{AB} in a complicated way. Since the number of the parameters is the same, it is straightforward to obtain the inverse relation, $G^{AB}=G^{AB}(g^2_{YM},g^{ab},C_{\hat{a}})$. This combination transforms linearly under $\mathrm{SL}(d+1,\mathbf{Z})$. In this sense, it is possible to claim that $\mathrm{SL}(d+1,\mathbf{Z})$ is a part of the U-duality symmetry and G^{AB} gives the parameter which transforms covariantly under $\mathrm{SL}(d+1,\mathbf{Z})$. The closure of these parameters under $\mathrm{SL}(d+1,\mathbf{Z})$ was discussed in the literature, for example, [20].

The parameters obtained from $\vec{\lambda}^A$, however, do not describe the full parameter space to implement U-duality. In the following, we compare it with the dimensions of the parameter space. As we see, for $d=1$, it correctly reproduces the moduli. The discrepancy of the number of parameters starts from $d>1$. We will explain some part of the missing parameters is given as the deformation of 3-algebra (2.9).

D3-brane $(d=1)$**.** It corresponds to M-theory compactified on T^2. The parameter space in this case is $\big(\mathrm{SL}(2)/\mathrm{U}(1)\big)\times\mathbf{R}$ which gives 3 scalars. They correspond to G^{00}, G^{01} and g, in other words, g^2_{YM}, $C_{\hat{1}}$ and g^{11}, all of which appear in the D3-brane action (2.65).

D4-branes $(d=2)$**.** It corresponds to M-theory compactified on T^3. The parameter space in this case is $\big(\mathrm{SL}(3)/\mathrm{SO}(3)\big)\times\big(\mathrm{SL}(2)/\mathrm{U}(1)\big)$ which gives 7 parameters. They correspond to G^{ab}, B_{ab}, Φ and $C_{\hat{a}}$ which transform in the $\mathbf{3}+\mathbf{1}+\mathbf{1}+\mathbf{2}$ representations of SL(2). Φ is dilaton which satisfies $e^\Phi=g_s=(2\pi)^{p-2}l_s^{p-3}g^2_{YM}$, and $C_{\hat{a}}$ is R-R 1-form (or $p-3$ form) field defined in eq. (3.14).

B_{ab} is NS-NS 2-form field which we have not discussed so far. As we commented in the footnote 4, such parameters were introduced in section 5.2 of [1] as the deformation of the 3-algebra, $[u_0,u_a,u_b]=B_{ab}T^0_{\vec{0}},\ \cdots$. It describes the noncommutativity on the torus along the line of [23]. We have not used this generalized algebra for the simplicity of the computation but can be straightwardly included in the L-BLG model. It is interesting that some part of moduli is described as dynamical variable ("Higgs VEV") while the other part comes from the modification of 3-algebra which underlies the L-BLG model.

D5-branes $(d=3)$**.** It corresponds to M-theory compactified on T^4. The parameter space in this case is $\mathrm{SL}(5)/\mathrm{SO}(5)$ which gives 14 parameters. They correspond to G^{ab}, B_{ab}, Φ, $C_{\hat{a}}$ and $C_{\hat{a}\hat{b}\hat{c}}$ which transform in the $\mathbf{6}+\mathbf{3}+\mathbf{1}+\mathbf{3}+\mathbf{1}$ representations of SL(3).

$C_{\hat{a}\hat{b}\hat{c}}:=C_{1\cdots\hat{a}\cdots\hat{b}\cdots\hat{c}\cdots d}$ is R-R 0-form (or $p-5$ form) field which causes the interaction like as $\epsilon^{\mu\nu\lambda 1\cdots d}C_{\hat{a}\hat{b}\hat{c}}F_{\mu\nu}F_{\lambda a}F_{bc}$ or $\epsilon^{\mu\nu\lambda 1\cdots d}C_{\hat{a}\hat{b}\hat{c}}F_{\mu a}F_{\nu b}F_{\lambda c}$. In the context of 3-algebra, there is a room to include such coupling [1]. It is related to the 3-algebra associated with Nambu-Poisson bracket. As shown in [5], the worldvolume theory becomes not the super Yang-Mills but instead described by self-dual 2-form field which describes the M5-brane.[6] The precise statement on the moduli becomes obscure in this sense.

To see U-duality, we must also consider the transformation of B_{ab} and $C_{\hat{a}\hat{b}\hat{c}}$. Especially, the interchange $B_{ab} \leftrightarrow C_{\hat{a}}$ and $C_{\hat{a}\hat{b}\hat{c}} \leftrightarrow \Phi$ means S-duality.

D6-branes ($d = 4$)**.** It corresponds to M-theory compactified on T^5. The parameter space in this case is $\mathrm{SO}(5,5)/\big(\mathrm{SO}(5) \times \mathrm{SO}(5)\big)$ which gives 25 scalars. They correspond to G^{ab}, B_{ab}, Φ, $C_{\hat{a}}$ and $C_{\hat{a}\hat{b}\hat{c}}$ which transform in the $\mathbf{10} + \mathbf{6} + \mathbf{1} + \mathbf{4} + \mathbf{4}$ representation of SL(4). To see U-duality, we must also consider the transformation of B_{ab} and $C_{\hat{a}\hat{b}\hat{c}}$.

D7-branes ($d = 5$)**.** It corresponds to M-theory compactified on T^6. The parameter space in this case is $E_6/\mathrm{USp}(8)$ which gives 42 scalars. They correspond to G^{ab}, B_{ab}, Φ, $C_{\hat{a}}$, $C_{\hat{a}\hat{b}\hat{c}}$ and $C_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}$ which transform in the $\mathbf{15}+\mathbf{10}+\mathbf{1}+\mathbf{5}+\mathbf{10}+\mathbf{1}$ representations of SL(5).

$C_{\hat{a}\hat{b}\hat{c}\hat{e}\hat{f}}$ is R-R 0-form (or $p-7$ form) field which causes the interaction like as $\epsilon^{\mu\nu\lambda 1\cdots d}$ $C_{\hat{a}\hat{b}\hat{c}\hat{e}\hat{f}} F_{\mu\nu} F_{\lambda a} F_{bc} F_{ef}$ and so on. Note that $C_{\hat{a}}$ in this case must be the self-dual 4-form field.

To see U-duality, we must also consider the transformation of B_{ab}, $C_{\hat{a}\hat{b}\hat{c}}$ and $C_{\hat{a}\hat{b}\hat{c}\hat{e}\hat{f}}$. Especially, the interchange $B_{ab} \leftrightarrow C_{\hat{a}\hat{b}\hat{c}}$ and $C_{\hat{a}\hat{b}\hat{c}\hat{e}\hat{f}} \leftrightarrow \Phi$ means S-duality. However we don't know the way to introduce the field $C_{\hat{a}\hat{b}\hat{c}\hat{e}\hat{f}}$ at this moment in time, so this discussion may be difficult.

D8-branes ($d = 6$)**.** It corresponds to M-theory compactified on T^7. The parameter space in this case is $E_7/\mathrm{SU}(8)$ which gives 70 scalars. They correspond to G^{ab}, B_{ab}, Φ, $C_{\hat{a}}$, $C_{\hat{a}\hat{b}\hat{c}}$ and $C_{\hat{a}\hat{b}\hat{c}\hat{e}\hat{f}}$ which transform in the $\mathbf{21} + \mathbf{15} + \mathbf{1} + \mathbf{6} + \mathbf{20} + \mathbf{6}$ representations of SL(6), plus one additional scalar B_{abcefg} which is the dual of NS-NS 2-form $*B_{(2)}$. To see U-duality, we must consider the transformation of all these fields.

D9-branes ($d = 7$)**.** It corresponds to M-theory compactified on T^8. The parameter space in this case is $E_8/\mathrm{SO}(16)$ which gives 128 scalars. They correspond to G^{ab}, B_{ab}, Φ, $C_{\hat{a}}$, $C_{\hat{a}\hat{b}\hat{c}}$, $C_{\hat{a}\hat{b}\hat{c}\hat{e}\hat{f}}$ and $C_{\hat{a}\hat{b}\hat{c}\hat{e}\hat{f}\hat{g}\hat{h}}$ which transform in the $\mathbf{28} + \mathbf{21} + \mathbf{1} + \mathbf{7} + \mathbf{35} + \mathbf{21} + \mathbf{1}$ representations of SL(7), plus 14 additional scalars B_{abcefg} and $C_{\mu a}$. This $C_{\mu a}$ is R-R 2-form field which has legs belong to one of worldvolume coordinates x^μ and one of torus coordinates y^a.

To see U-duality, we must consider the transformation of all these fields. However we don't know the way to introduce the field $C_{\hat{a}\hat{b}\hat{c}\hat{e}\hat{f}}$ and $C_{\hat{a}\hat{b}\hat{c}\hat{e}\hat{f}\hat{g}\hat{h}}$ at this moment in time, so this discussion may be very difficult.

4 Conclusion and discussion

In this paper, we have presented a detailed derivation of Dp-brane action from BLG model. The VEV's of ghost fields $\vec{\lambda}^A$ give the moduli of torus T^d ($d = p-2$) g^{ab}, the coupling constants g_{YM} of super Yang-Mills and the R-R $(p-3)$-form field $C_{\hat{a}}$ through the 'metric' $G^{AB} = \vec{\lambda}^A \cdot \vec{\lambda}^B$. For D3-branes ($d = 1$), the parameters thus obtained are enough to

[6]In order to satisfy the fundamental identity, Nambu-Poisson bracket must be equipped on a 3-dimensional manifold. So, in this case, we must choose the specific T^3 where Nambu-Poisson bracket is defined from the whole compactified torus T^4.

realize full Montonen-Olive duality group SL$(2,\mathbf{Z})$ through the linear transformation on $\vec{\lambda}^A$. Moreover, some part of the symmetry is actually the automorphism of 3-algebra. For higher dimensional case $d > 1$ (Dp-branes with $p > 3$), these parameters are enough to implement a subgroup of U-duality transformation, SL$(d+1,\mathbf{Z})$, which acts linearly on $\vec{\lambda}^A$. The transformations of various parameters can be determined through the linear transformation of the metric G^{AB}. In order to realize the full U-duality group, however, they are not enough. We argue that one of the missed parameters, NS-NS 2-form background, can be introduced through the deformation of the 3-algebra. For higher d, we need extra R-R background which we could not succeed to explain in the context of L-BLG models so far. One possibility may be to use the coupling constants of Nambu-Poisson bracket which gives rise to self-dual 2-form field on the worldvolume instead of super Yang-Mills.

There are a few directions for the futher development from current work. One direction is to understand the higher d case in more detail. For higher d, we have to think more carefully on the fundamental degree of freedom. In some cases, the gauge theory should be replaced by 2-form fields, and sometimes by strings. We hope that the BLG description of M5-brane [5] gives an essential hint.

It is also interesting to derive the U-duality symmetry from ABJM model. While some work have been done in [14] for D3-brane, it may be interesting how to incorporate the loop algebras in ABJM context which would help us to go beyond D3. As we explained here, the loop algebra is suitable symmetry to describe the Kaluza-Klein modes.

Another interesting direction is to describe the curved background or D-branes from L-BLG model. As we already explained in the text, as long as we start from BLG model, we arrive at a flat background. This is natural since we have maximal supersymmetry. If, however, one modifies the action slightly (a naive discussion is given in the text), there is more room to incorporate various degrees of freedom. Such modification of the model seems essential to understand various M-brane dynamics.

Acknowledgments

We would like to thank Pei-Ming Ho for sharing his excellent insights through the collaboration at some stages. Y. M. is partially supported by KAKENHI (#20540253) from MEXT, Japan. S. S. and T. K. are partially supported by Global COE Program "the Physical Sciences Frontier", MEXT, Japan.

A SL$(d+1, \mathbf{Z})$ transformations on Dp-branes

In this appendix, we compute the transformation law for the moduli parameters under SL$(d+1,\mathbf{Z})$ transformation (3.15). SL$(d+1,\mathbf{Z})$ is generated by the following two kinds of

$(d+1)\times(d+1)$ matrices:

$$S(i,j)\,:\,\begin{cases} \Lambda^A{}_B = \delta^A_B \quad (\text{for } A,B\neq i,j)\,, \\ \begin{pmatrix} \Lambda^i{}_i & \Lambda^i{}_j \\ \Lambda^j{}_i & \Lambda^j{}_j \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \end{cases}$$

$$T(i,j;n)\,:\,\begin{cases} \Lambda^A{}_B = \delta^A_B \quad (\text{for } A,B\neq i,j)\,, \\ \begin{pmatrix} \Lambda^i{}_i & \Lambda^i{}_j \\ \Lambda^j{}_i & \Lambda^j{}_j \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}. \end{cases}$$

where $i,j=0,1,\ldots,d$ $(i<j)$ and $n\in\mathbf{Z}$. Obviously, $S(i,j)$ is a generalization of S-duality transformation and $T(i,j;n)$ is the generalization of translation generator.

(I) $\Lambda = S(0,i)$ $(i\neq 0)$**.** This transformation interchanges $\vec{\lambda}^0$ and $\vec{\lambda}^i$, i.e. M-direction and one of the torus directions. It is a generalization of S-duality transformation for $d=1$ case. G^{0A} and g^{ab} are transformed as

$$\begin{aligned} G'^{00} &= G^{ii}\,, & G'^{0i} &= -G^{0i}\,, & G'^{0a} &= G^{ia}\,, \\ g'^{ii} &= g^{ii}, & g'^{ia} &= -(G^{ii}G^{0a}-G^{i0}G^{ia})\,, & g'^{ab} &= G^{ii}G^{ab}-G^{ia}G^{ib}\,, \end{aligned} \tag{A.1}$$

for $a,b\neq 0,i$. In the simple case of $G^{0a}=G^{0i}=G^{ia}=0$,

$$g^2_{YM} = \sqrt{\frac{G^{00}}{G^{ii}}}\,\frac{(2\pi)^d}{(G^{00})^{(d-1)/2}}\,\frac{1}{\sqrt{\hat{G}^i}} \quad\to\quad g'^2_{YM} = \sqrt{\frac{G^{ii}}{G^{00}}}\,\frac{(2\pi)^d}{(G^{ii})^{(d-1)/2}}\,\frac{1}{\sqrt{\hat{G}^i}}\,, \tag{A.2}$$

where $\hat{G}^i$ is the minor determinant of G^{ab} excluding the i'th row and column. On the other hand, $C_{\hat{a}}$ remains zero in this simple case.

(II) $\Lambda = T(0,i;n)$ $(i\neq 0)$**.** This transformation shifts the direction as $\vec{\lambda}^0\to\vec{\lambda}^0$ and $\vec{\lambda}^i\to\vec{\lambda}^i+n\vec{\lambda}^0$, and should be a generalization of T-duality transformation. G^{0A} and g^{ab} are transformed as

$$\begin{aligned} G'^{00} &= G^{00}\,, & G'^{0i} &= G^{0i}+nG^{00}\,, & G'^{0a} &= G^{0a}\,, \\ g'^{ii} &= g^{ii}\,, & g'^{ia} &= g^{ia}\,, & g'^{ab} &= g^{ab}\,, \end{aligned} \tag{A.3}$$

for $a,b\neq 0,i$. So the coupling constant g^2_{YM} is invariant under this transformation. On the other hand, one component of R-R field $C_{(d-1)}$ is shifted as in the D3-branes case,

$$C_{\hat{i}} \;\to\; C'_{\hat{i}} = C_{\hat{i}} + \frac{n}{4(2\pi)^d(d-1)!}\,\frac{\sqrt{g}}{\sqrt{g^{ii}}}\,, \tag{A.4}$$

while all the other components remain the same.

(III) $\Lambda = S(i,j)$ $(i,j \neq 0)$**.** This transformation interchanges $\vec{\lambda}^i$ and $\vec{\lambda}^j$ and should make no physical change. In fact,

$$\begin{aligned} G'^{00} &= G^{00}\,, \qquad G'^{0i} = G^{0j}\,, \qquad G'^{0j} = -G^{0i}\,, \\ g'^{ii} &= g^{jj}\,, \qquad g'^{ij} = -g^{ji}\,, \qquad g'^{ji} = -g^{ij}\,, \qquad g'^{jj} = g^{ii}\,, \end{aligned} \tag{A.5}$$

and other G^{0a} and g^{ab} remain the same. The coupling constant g_{YM}^2 is invariant under this transformation. The components of $C_{(d-1)}$ is shuffled by the interchange of the basis $\{\vec{\lambda}^a\}$, but this doesn't mean any physical changes.

(IV) $\Lambda = T(i,j;n)$ $(i,j \neq 0)$**.** This transformation shifts the torus direction as $\vec{\lambda}^i \to \vec{\lambda}^i$ and $\vec{\lambda}^j \to \vec{\lambda}^j + n\vec{\lambda}^i$. In this case, G^{0A} and g^{ab} are transformed as

$$\begin{aligned} G'^{00} &= G^{00}\,, & G'^{0j} &= G^{0j} + nG^{0i}\,, & G'^{0a} &= G^{0a}\,, \\ g'^{jj} &= g^{jj} + 2ng^{ji} + n^2 g^{ii}\,, & g'^{ja} &= g^{ja} + ng^{ia}\,, & g'^{ab} &= g^{ab}\,, \end{aligned} \tag{A.6}$$

for $a,b \neq 0,j$. Since $\sqrt{g}$ (or the volume of T^d) remains the same, g_{YM}^2 is invariant under this transformation. The components of $C_{(d-1)}$, just as in the case of $S(i,j)$, is effected by the transformation of the basis $\{\vec{\lambda}^a\}$, but it is not physically meaningful.

As we discussed in section 3.2, the transformation laws are somewhat complicated, since the parameters g_{YM}^2 and $C_{\hat{a}}$ depends on G^{00} and G^{0a} in complicated way. So if we want to see concisely the correspondence between subgroup of U-duality $\mathrm{SL}(d+1,\mathbf{Z})$ and transformation of VEV's (3.15), we must notice the transformation of $G^{AB} = \vec{\lambda}^A \cdot \vec{\lambda}^B$. In fact, G^{AB} is the linear realization of $\mathrm{SL}(d+1,\mathbf{Z})$ transformation (3.15), and the parameters g_{YM}^2 and $C_{\hat{a}}$ transform complexly through this covariant transformation of G^{AB}.

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